Intuitionistic Fuzzy αg^{**}-Closed sets, Intuitionistic Fuzzy αg^{**}-Continuity and Intuitionistic Fuzzy αg^{**}- Homeomorphisms

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Abstract:
In this paper, we introduce and study the notions of intuitionistic fuzzy αg^{**}*- closed sets, intuitionistic fuzzy* αg^{**}*-continuity, intuitionistic fuzzy* αg^{**}*- open mapping, intuitionistic fuzzy* αg^{**}*- closed mapping, intuitionistic fuzzy* αg^{**}*- homeomorphisms and some of its properties in intuitionistic fuzzy topological spaces.*

Key words: *Intuitionistic fuzzy* g^{*}*- closed set, intuitionistic fuzzy* g^{**}*- closed sets, intuitionistic fuzzy* αg^{**}*- closed sets, intuitionistic fuzzy* αg^{**}*- continuity, intuitionistic fuzzy* αg^{**}*- open mapping, intuitionistic fuzzy* αg^{**}*- closed mapping and intuitionistic fuzzy* αg^{**}*- homeomorphisms*

1. Introduction

Zadeh [12] introduced the notion of fuzzy sets. Later on, fuzzy topology was introduced by Chang [2] in 1967.The concept of intuitionistic fuzzy topology was introduced by Atanassov [1] as a generalization of fuzzy sets.Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological space. In this paper, we introduce the concepts of intuitionistic fuzzy αg^{**}-closed sets, intuitionistic fuzz αg^{**}-continuity, intuitionistic fuzzy αg^{**}-open mapping, intuitionistic fuzzy αg^{**}-closed mapping, intuitionistic fuzzy αg^{**}-homeomorphisms and study some of its properties in intuitionistic fuzzy topological spaces.

2. Preliminaries

2.1 Definition [1]

An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$, where the functions $\mu_A: X \to [0,1]$ and $\gamma_A: X \to [0,1]$ denote the the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A respectively and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

2.2 Definition [1]

Let A and B be intuitionistic fuzzy sets of the form $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ and $B = \{< x, \mu_B(x), \gamma_B(x) >: x \in X\}$. Then

1. $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$.

2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $A^c = \{< x, \gamma_A(x), \mu_A(x) >: x \in X\}$.
4. $A \cap B = \{< x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) >: x \in X\}$.
5. $A \cup B = \{< x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) >: x \in X\}$.

For the sake of simplicity, we shall use the notation $A =< x, \mu_A, \gamma_A >$ instead of $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$. Also for the sake of simplicity, we shall use the notation $A =< x, (\mu_A, \mu_B), (\gamma_A, \gamma_B) >$ instead of $A =< x, (A/\mu_A, B/\mu_B), (A/\gamma_A, B/\gamma_B) >$.

The intuitionistic fuzzy sets $0_\sim = \{< x, 0, 1 >: x \in X\}$ and $1_\sim = \{< x, 1, 0 >: x \in X\}$ are respectively the empty and whole set of X.

2.3 Definition [3]

An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

1. $0_\sim, 1_\sim \in \tau$,
2. $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
3. $\cup G_i \in \tau$, for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic closed set (IFCS in short) in X.

2.4 Definition [3]

Let (X, τ) be an IFTS and $A =< x, \mu_A, \gamma_A >$ be an IFS in X. Then

1. int(A) = $\cup \{G / G$ is an IFOS in X and $G \subseteq A\}$.
2. cl(A) = $\cap \{K / K$ is an IFCS in X and $A \subseteq K\}$.
3. cl(A^c) = (int(A))^c.
4. int(A^c) = (cl(A))^c.

2.5 Result [9]

Let A be an IFS in (X, τ). Then

1. $\alpha cl(A) = A \cup cl(int(cl(A)))$
2. $\alpha int(A) = A \cap int(cl(int(A)))$

2.6 Definition [4]

An IFS $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ in an IFTS (X, τ) is said to be an

1. intuitionistic fuzzy regular open set (IFROS) if $A = int(cl(A))$.
2. intuitionistic fuzzy α- open set (IFαOS) if $A \subseteq int(cl(int(A)))$.

An IFS A is said to be an intuitionistic fuzzy regular closed set (IFRCS) and intuitionistic fuzzy α- closed set (IFαCS) if the complement of A is an IFROS and IFαOS respectively.

2.7 Definition

An IFS $A = \{< x, \mu_A(x), \gamma_A(x) >: x \in X\}$ in an IFTS (X, τ) is said to be an

1. Intuitionistic fuzzy α- generalized closed set (IFαGCS) [6] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.
2. Intuitionistic fuzzy regular generalized closed set (IFRGCS) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

An IFS A is said to be an intuitionistic fuzzy α- generalized open set (IFαGOS) and intuitionistic fuzzy regular generalized open set (IFRGOS) if the complement of A is IFαGCS and IFRGCS respectively.

2.8 Definition [4]
Let (X,τ) and (Y,σ) be two intuitionistic fuzzy topological spaces and let $f: X \rightarrow Y$ be a function. Then f is said to be an intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of Y is an intuitionistic fuzzy open set in X.

2.9 Definition [9]
Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an intuitionistic fuzzy α-continuous if $f^{-1}(B) \in$ IFαO(X) for every $B \in \sigma$.

2.10 Definition [7]
Let (X,τ) and (Y,σ) be two intuitionistic fuzzy topological spaces and let $f: X \rightarrow Y$ be a function. Then f is said to be an intuitionistic fuzzy αg-continuous if pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy αg-closed in X.

2.11 Definition [11]
Let $\alpha, \beta \in [0,1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (briefly IFP), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta), & if\ x = P \\ (0,1) & otherwise. \end{cases}$$

We observe that an IFP $p_{(\alpha,\beta)}$ is said to belong to an IFS $A = < x, \mu_A(x), \gamma_A(x) >$, denoted by $p_{(\alpha,\beta)} \in A$ if $\alpha \leq \mu_A(x)$ and $\beta \geq \gamma_A(x)$.

2.12 Definition [11]
Two IFSs A and B are said to be q-coincident ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \gamma_B(x)$ or $\gamma_A(x) < \mu_B(x)$.

2.13 Definition [11]
Two IFSs are said to be not q-coincident ($A_q^c B$ in short) if and only if $A \subseteq B^c$.

2.14 Definition [4]
Let (X,τ) and (Y,σ) be two intuitionistic fuzzy topological spaces and let $f: X \rightarrow Y$ be a function. Then f is said to be an

(i) Intuitionistic fuzzy closed map if the image of each intuitionistic fuzzy closed set in X is an intuitionistic fuzzy closed set in Y.
(ii) Intuitionistic fuzzy open map if the image of each intuitionistic fuzzy open set in X is an intuitionistic fuzzy open set in Y.

2.15 Definition [5]
Let f be a mapping from an IFTS (X,τ) into an IFTS(Y,σ). Then f is said to be an intuitionistic fuzzy α-closed mapping (IFα-closed mapping in short) if $f(A)$ is an IFαCS in Y for every IFCS A in X.

2.16 Definition [8]
Let f be a bijection mapping from an IFTS (X,τ) into an IFTS(Y,σ). Then f is said to be an

(i) Intuitionistic fuzzy homeomorphism (IF homeomorphism in short) if f and f^{-1} are IF continuous mappings.
(ii) Intuitionistic fuzzy α-homeomorphism (IFα-homeomorphism in short) if f and f^{-1} are IFα- continuous mappings.

3. Intuitionistic Fuzzy αg^{**}- Closed sets

In this section, we introduced the concept of intuitionistic fuzzy αg^{**}-closed sets and studied some of its properties in intuitionistic fuzzy topological spaces.

3.1 Definition An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy g^*-closed set (briefly IFg^*CS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFGOS in X.

3.2 Example Let X = {a, b} and $\tau = \{0_{\sim}, A, 1_{\sim}\}$ be an IFTS on X, where
A =<x, (0.5,0.6), (0.3,0.2)>. Then S = <x, (0.2,0.2), (0.6,0.6)> is an IFg^*CSin X.

3.3 Definition An IFS A of an IFTS (X,τ) is said to be an intuitionistic fuzzy g^*-open set (briefly IFg^*OS) if the complement A^c is an IFg^*CS in X.

3.4 Example Let X = {a, b} and $\tau = \{0_{\sim}, A, 1_{\sim}\}$ be an IFTS on X, where
A =<x, (0.5,0.6), (0.3,0.2)>. Then S = <x, (0.5,0.6), (0.3,0.2)> is an IFg^*OSin X.

3.5 Definition An IFS A of an IFTS (X,τ) is said to be an intuitionistic fuzzy g^{**}-closed set (briefly IFg^{**}CS) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFg^*OS in X.

3.6 Example Let X = {a, b} and $\tau = \{0_{\sim}, A, 1_{\sim}\}$ be an IFTS on X, where
A =<x, (0.5,0.6), (0.3,0.2)>. Then S = <x, (0.2,0.2), (0.6,0.7)> is an IFg^{**}CSin X.

3.7 Definition An IFS A of an IFTS (X,τ) is said to be an intuitionistic fuzzy g^{**}-open set (briefly IFg^{**}OS) if the complement A^c is an IFg^{**}CS in X.

3.8 Example Let X = {a, b} and $\tau = \{0_{\sim}, A, 1_{\sim}\}$ be an IFTS on X, where
A =<x, (0.5,0.6), (0.3,0.2)>. Then S = <x, (0.5,0.6),(0.3,0.2)> is an IFg^{**}OSin X.

3.9 Theorem Every IFCS (resp. IFOS) is an IFg^{**}CS (resp. IFg^{**}OS) but not conversely.
Proof: Let A⊆U and U is IFOS in (X,τ). Since $\alpha cl(A) \subseteq cl(A)$ and A is an IFCS, $\alpha cl(A) \subseteq cl(A) = A \subseteq U$. Therefore A is an IF g^{**}CS in X.

3.10 Example Let X = {a, b} and $\tau = \{0_{\sim}, A, 1_{\sim}\}$ be an IFTS on X, where
A=<x, (0.2,0.4), (0.1,0.2)>. Then S = <x, (0.1,.2), (.4,0.5)> is an IFg^{**}CS but not IFCS in X.

3.11 Definition An IFS A of an IFTS (X, τ) is said to be intuitionistic fuzzy αg^{**}-closed set (briefly IFαg^{**}CS) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is IFg^{**}OS in X. The set of all IFαg^{**}CSs of X is denoted by IFαg^{**}C(X).

3.12 Example Let X = {a, b} and $\tau = \{0_{\sim}, A, 1_{\sim}\}$ be an IFTS on X, where
A= < x, (0.5, 0.6), (0.3, 0.2) >. Then the IFS S= < x, (0.3, 0.2), (0.5, 0.6) > is an IFα^{**}GCS in (X, τ).

3.13 Theorem Every IFCS, IFRCS and IFαCS in (X, τ) is an IFαg^{**}CS, but not conversely.
Proof: *Obvious*

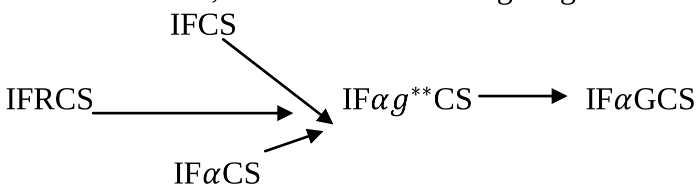
3.14 Example (i) Let X = {a, b} and $\tau = \{0_{\sim}, A, 1_{\sim}\}$ be an IFTS on X where A= < x, (0.5, 0.6), (0.3, 0.2) >. Then S= < x, (0.2, 0.2), (0.5, 0.6) > is an IFαg^{**}CS but not an IFCS and IFRCS in X.

(ii) Let X = {a, b} and $\tau = \{0_\sim, A, 1_\sim\}$ be an IFTS on X where A= < x, (0.2, 0.1), (0.2, 0.4) >. Then S= < x, (0.2, 0.2), (0.6, 0.8) > is an IFαg^{**}CS but not an IFαCS in X.

3.15 Theorem Every IFαg^{**}CS in (X, τ) is an IFαGCS, but not conversely.
Proof: Let $A \subseteq U$ where U is IFOS in X. Since every IFOS is an IFg^{**}OS and A is IFαg^{**}CS we have $\alpha cl(A) \subseteq U$. Hence A is an IFαGCS in X.

3.16 Example Let X = {a, b} and $\tau = \{0_\sim, A, 1_\sim\}$ be an IFTS on X, where A= < x, (0.5, 0.4), (0.2, 0.1) >. Then the IFS S= < x, (0.6, 0.3), (0.2, 0.1) > is an IFαGCS in (X, τ) but not an IFαg^{**}CS in X.

3.17 Remark
From the above theorems, we have the following diagram.



where $A \rightarrow B$ represents A imply B, $A \nrightarrow B$ represents A does not imply B.

3.18 Theorem Union of two IFαg^{**}CSs again an IFαg^{**}CS.
Proof: Let U be an IFg^{**}OS in X, such that$A \cup B \subseteq U$. Since A and B are IFαg^{**}CSs we have $\alpha cl(A) \subseteq U$ and $\alpha cl(B) \subseteq U$. Therefore $\alpha cl(A) \cup \alpha cl(B) \subseteq \alpha cl(A \cup B) \subseteq U$. Hence $A \cup B$ is an IFαg^{**}CS in (X, τ).

3.19 Theorem If A is an IFαg^{**}CS and $A \subseteq B \subseteq \alpha cl(A)$ then B is an IFαg^{**}CS.
Proof: Let U be an IFg^{**}OS such that $\subseteq U$. Since A is an IFαg^{**}CS, we have $\alpha cl(A) \subseteq U$. By hypothesis $B \subseteq \alpha cl(A)$ then $\alpha cl(B) \subseteq \alpha cl(A)$. This implies $\alpha cl(B) \subseteq U$. Hence B is an IFαg^{**}CS.

3.20 Definition An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzyαg^{**} open set (IFαg^{**}OS in short) if and only if A^c is an IFαg^{**}CS in (X, τ).

3.21 Theorem For any IFTS (X, τ), we have the following:
- (i) Every IFOS is an IFαg^{**}OS
- (ii) Every IFαOS is an IFαg^{**}OS
- (iii) Every IFROS is an IFαg^{**}OS

Proof: Obvious.

3.21 Theorem If A is an IFαg^{**}OS and $\alpha int(A) \subseteq B \subseteq A$, then B is an IF$\alpha g^{**}$OS.
Proof: If $\alpha int(A) \subseteq B \subseteq A$, then $A^c \subseteq B^c \subseteq (\alpha int(A))^c = \alpha cl(A^c)$. Since A^c is an IFαg^{**}CS, then by Theorem 3.21, B^c is an IFαg^{**}CS. Therefore B is an IFαg^{**}OS.

4. Intuitionistic Fuzzy $\alpha^{**}g$- Continuity

In this section we introduced the concept of intuitionistic fuzzy $\alpha^{**}g$ -continuous mapping and studied some of its properties.

4.1 Definition A mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ is an intuitionistic fuzzy αg^{**} -continuous (briefly IFαg^{**} -continuous) if inverse image of every intuitionistic fuzzy closed set of Y is an intuitionistic fuzzy αg^{**}-closed set in X.

4.2 Example Let X= {a, b} , Y={u, v} and A= < x, (0.1, 0.2), (0.2, 0.2) >, B = < y, (0.5,0.6), (0.1,0.1) >. Then $\tau = \{0_{\sim}, A, 1_{\sim}\}$, $\sigma = \{0_{\sim}, B, 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by $\mathrm{f(a) = u}$ and $\mathrm{f(b) = v}$. Then f is an IFαg^{**} -continuous mapping.

4.3 Theorem A mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ is an IFαg^{**} -continuous if and only if the inverse image of every IFOS of Y is an IFαg^{**}OS in X.
Proof: It is obvious, because $f^{-1}(B^c) = (f^{-1}(B))^c$ for every IFS B of Y.

4.4 Theorem Every intuitionistic fuzzy continuous mapping is IFαg^{**}- continuous, but converse may not be true.
Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ is an intuitionistic fuzzy continuous mapping. Let A be an IFCS in Y. Then $f^{-1}(A)$ is IFCS. Since every IFCS is IFαg^{**}CS , $f^{-1}(A)$ is an IFαg^{**}CS in X. Hence f is an IFαg^{**}-continuous mapping.

4.5 Example Let X= {a, b} , Y={u, v} and A= < x, (0.5, 0.6), (0.3, 0.2) >, B = < y, (0.5,0.6), (0.2,0.2) >. Then $\tau = \{0_{\sim}, A, 1_{\sim}\}$, $\sigma = \{0_{\sim}, B, 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by $\mathrm{f(a) = u}$ and $\mathrm{f(b) = v}$.The intuitionistic fuzzy set S = < y, (0.2, 0.2), (0.5, 0.6) > is IFCS in Y. Then $f^{-1}(S)$ is IFαg^{**}CS in X but not IFCS in X. Therefore, f is an IFαg^{**} -continuous mapping but not an intuitionistic fuzzy continuous mapping.

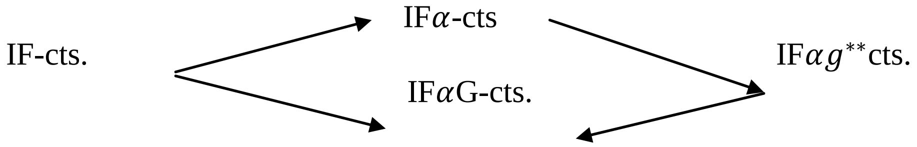
4.6 Theorem Every intuitionistic fuzzy α-continuous mapping is IFαg^{**}-continuous, but converse may not be true.
Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ is an intuitionistic fuzzy α-continuous mapping. Let A be an IFCS in Y. Then $f^{-1}(A)$ is IFαCS. Since every IFαCS is IFαg^{**}CS , $f^{-1}(A)$ is an IFαg^{**}CS in X. Hence f is an IFαg^{**}-continuous mapping.

4.7 Example Let X= {a, b} , Y={u, v} and A= < x, (0.2, 0.1), (0.2, 0.4) >, B = < y, (0.2,0.4), (0.2,0.1) >. Then $\tau = \{0_{\sim}, A, 1_{\sim}\}$, $\sigma = \{0_{\sim}, B, 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by $\mathrm{f(a) = u}$ and $\mathrm{f(b) = v}$.The intuitionistic fuzzy set S = < y, (0.2, 0.1), (0.2, 0.4) > is IFCS in Y. Then $f^{-1}(S)$ is IFαg^{**}CS in X but not IF αCS in X. Therefore, f is an IFαg^{**} -continuous mapping but not an intuitionistic fuzzy α-continuous mapping.

4.8 Theorem Every intuitionistic fuzzy αg^{**}-continuous mapping is IFαG-continuous, but converse may not be true.
Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ is an intuitionistic fuzzy αg^{**}-continuous mapping. Let A be an IFCS in Y. By hypothesis, $f^{-1}(A)$ is an IFαGCS in X. Hence f is an IFαG -continuous mapping.

4.9 Example Let X= {a, b} , Y={u, v} and A= < x, (0.5, 0.4), (0.2, 0.1) >, B = < y, (0.2,0.1), (0.6,0.3) >. Then $\tau = \{0_{\sim}, A, 1_{\sim}\}$, $\sigma = \{0_{\sim}, B, 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by $\mathrm{f(a) = u}$ and $\mathrm{f(b) = v}$.The intuitionistic fuzzy set S = < y, (0.6, 0.3), (0.2, 0.1) > is IFCS in Y.

Then $f^{-1}(S)$ is IFαGCS in X but not IFαg^{**}CS in X. Therefore f is an IFαG -continuous mapping but not an intuitionistic fuzzy αg^{**}-continuous mapping.
The relation between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram 'cts.' means continuous.



4.10 Theorem If $f: (X,\tau) \rightarrow (Y,\sigma)$ is an IFαg^{**} -continuous then for each IFP $p_{(\alpha,\beta)}$ of X and each IFOS B of Y such that $f\left(p_{(\alpha,\beta)}\right) \subseteq B$ there exists an intuitionistic fuzzy αg^{**}-open set A of X such that $p_{(\alpha,\beta)} \subseteq A$ and $f(A) \subseteq B$.
Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and B be an IFOS of Y such that$f\left(p_{(\alpha,\beta)}\right) \subseteq B$. Put $A = f^{-1}(B)$. Then by hypothesis A is an intuitionistic fuzzy αg^{**}-open set of X such that $p_{(\alpha,\beta)} \subseteq A$ and $f(A) = f\left(f^{-1}(B)\right) \subseteq B$.

4.11 Theorem If $f: (X,\tau) \rightarrow (Y,\sigma)$ is an IFαg^{**} -continuous then for each IFP $p_{(\alpha,\beta)}$ of X and each IFOS B of Y such that $f\left(p_{(\alpha,\beta)}\right)_q B$ there exists an intuitionistic fuzzy αg^{**}-open set A of X such that $p_{(\alpha,\beta)}{}_q A$ and $f(A) \subseteq B$.
Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and B be an IFOS of Y such that $f\left(p_{(\alpha,\beta)}\right)_q B$. Put $A = f^{-1}(B)$. Then by hypothesis A is an intuitionistic fuzzy αg^{**}-open set of X such that $p_{(\alpha,\beta)}{}_q A$ and $f(A) = f\left(f^{-1}(B)\right) \subseteq B$.

4.12 Definition Let (X,τ) be an IFTS and A be an IFS in X. Then αg**- interior and αg**-closure of A are defined as

$$\alpha g^{**}cl(A) = \cap \{\text{K: K is an IF}\alpha g^{**}\text{CS in X and } A \subseteq K\}$$
$$\alpha g^{**}int(A) = \cup \{\text{G: G is an IF}\alpha g^{**}\text{OS in X and } G \subseteq A\}$$

If A is IFαg^{**}CS, then $\alpha g^{**}cl(A) = A$.

4.13 Theorem If $f: (X,\tau) \rightarrow (Y,\sigma)$ is IF αg^{**}- continuous and $g: (Y,\sigma) \rightarrow (Z,\mu)$ in intuitionistic fuzzy continuous, then $g \circ f: (X,\tau) \rightarrow (Z,\mu)$ is IFαg^{**} - continuous.
Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y because g is intuitionistic fuzzy continuous. Therefore, $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is an IFαg^{**}CS in X. Hence $g \circ f$ is IFαg^{**}- continuous.

4.14 Theorem Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be a mapping and let $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y. Then f is an IFαg^{**} -continuous mapping.
Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFαg^{**}CS, $f^{-1}(A)$ is an IFαg^{**}CS in X. Hence f is an IFαg^{**} -continuous mapping.

4.15 Theorem Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be an IFαg^{**} - continuous mapping. Then the following conditions are hold:

(i) $f(\alpha g^{**}cl(A)) \subseteq cl(f(A))$, for every IFS A in X
(ii) $\alpha g^{**}cl\left(f^{-1}(B)\right) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

Proof: (i) Since $cl(f(A))$ is an IFCS in Y and f is an IFαg^{**} -continuous mapping, then $f^{-1}(cl(f(A)))$ is IFαg^{**}CS in X. That is $\alpha g^{**}cl(A) \subseteq f^{-1}(cl(f(A)))$. Therefore

$f(\alpha g^{**}cl(A)) \subseteq cl(f(A))$, for every IFS A in X. **(ii)** Replacing A by $f^{-1}(B)$ in (i), we have $f\left(\alpha g^{**}cl(f^{-1}(B))\right) \subseteq cl\left(f(f^{-1}(B))\right) \subseteq cl(B)$. Hence $\alpha g^{**}cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

5. Intuitionistic Fuzzy αg^{**}- Open Mapping

In this section we introduced the concept of intuitionistic fuzzy αg^{**}- open mapping and studied some of its properties.

5.1 Definition A mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ is an intuitionistic fuzzy αg^{**}-open mapping (briefly IFαg^{**} -open mapping) if the image of every IFOS in X is IFαg^{**}OS in Y.

5.2 Example Let X = {a, b}, Y = {x, y} and A= < x, (0.2, 0.1), (0.6, 0.3) >, B = < y, (0.2, 0.1), (0.5, 0.2) >. Then $\tau = \{0_\sim, A, 1_\sim\}$, $\sigma = \{0_\sim, B, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by f(a) = x and f(b) = y. Then f is an IFαg^{**} -open mapping.

5.3 Theorem Every intuitionistic fuzzy open map is an IFαg^{**} -open map but converse may not be true.
Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic fuzzy open mapping. Let A be an IFOS in X. Since f is an intuitionistic fuzzy open mapping, $f(A)$ is an IFOS in Y. Since every IFOS is an IFαg**OS, $f(A)$ is an IFα^{**}GOS in Y. Hence f is an IFαg^{**} -open mapping.

5.4 Example Let X = {a, b}, Y = {x, y} and A= < x, (0.2, 0.1), (0.6, 0.3) >, B = < y, (0.2, 0.1), (0.5, 0.2) >. Then $\tau = \{0_\sim, A, 1_\sim\}$, $\sigma = \{0_\sim, B, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by f(a) = x and f(b) = y. Then f is an IFαg^{**} -open mapping but not an intuitionistic fuzzy open mapping .

5.5 Theorem $f:(X,\tau) \rightarrow (Y,\sigma)$ is IFαg^{**} - open map then $int(f^{-1}(G)) \subseteq f^{-1}(\alpha g^{**}int(G))$ for every IFS G of Y.
Proof: Let G be an IFS of Y. Then $int(f^{-1}(G))$ is an IFOS in X. Since f is an IFαg^{**} -open map $f(int(f^{-1}(G)))$ is IFαg**OS in Y and hence $f(int(f^{-1}(G))) \subseteq \alpha g^{**}int(f(f^{-1}(G))) \subseteq \alpha g^{**}int\ (G)$. Thus $int(f^{-1}(G)) \subseteq f^{-1}(\alpha g^{**}int(G))$.

5.6 Theorem A mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ is an IFαg^{**} -open if and only if for each IFS S of Y and for each IFCS U of X containing $f^{-1}(S)$ there is an IFαg^{**}CS V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.
Proof:
Necessity: Suppose that f is an IFαg^{**} -open map. Let S be the IFCS of Y and U be an IFCS of X such that $f^{-1}(S) \subseteq U$. Then $V = (f^{-1}(U^c))^c$ is an IFαg^{**}CS of Y such that $f^{-1}(V) \subseteq U$.
Sufficiency: Suppose that F is an IFOS of X. Then $f^{-1}(f(F))^c \subseteq F^c$ and F^c is an IFCS in X. By hypothesis there is an IFαg^{**}CS V of Y such that $(f(F))^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$, since V^c is an $IF\alpha g^{**}OS$ of Y. Hence $f(F)$ is an IFαg^{**}OS in Y and thus f is an IFαg^{**} -open map.

5.7 Theorem A mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ is an IFαg^{**} -open if and only if $f^{-1}(\alpha g^{**}cl(B)) \subseteq cl(f^{-1}(B))$ for every IFS B of Y.

Proof:
Necessity: Suppose that f is an IFαg^{**} -open map. For any IFS B of Y, $f^{-1}(B) \subseteq cl(f^{-1}(B)$. Therefore by Theorem (5.6) there exists an IFαg^{**}CS F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq cl(f^{-1}(B)$. Therefore we obtain that $f^{-1}\left(\alpha g^{**}cl(B)\right) \subseteq f^{-1}(F) \subseteq cl(f^{-1}(B))$.
Sufficiency: Suppose that B is an IFS of Y and F is an IFCS of X containing $f^{-1}(B)$. Put $V = cl(B)$, then $B \subseteq V$ and V is IFαg^{**}CS and $f^{-1}(V) \subseteq cl(f^{-1}(B) \subseteq F$. Then by Theorem (5.6) f is an IFαg^{**} -open map.

6. Intuitionistic Fuzzy αg^{**}- Closed Mapping

In this section we introduced the concept of intuitionistic fuzzy αg^{**}- closed mapping and studied some of its properties.

6.1 Definition A mapping $f: (X, \tau) \to (Y, \sigma)$ is an intuitionistic fuzzy αg^{**}-closed mapping (briefly IFαg^{**}-closed mapping) if the image of every IFCS in X is IFαg^{**}CS in Y.

6.2 Example Let X = {a, b}, Y = {x, y} and A= < x, (0.5, 0.6), (0.2, 0.1) >, B = < y, (0.5, 0.4), (0.2, 0.1) >. Then $\tau = \{0_\sim, A, 1_\sim\}$, $\sigma = \{0_\sim, B, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = x and f(b) = y. Then f is an IFαg^{**} -closed mapping.

Theorem 6.3: Every intuitionistic fuzzy closed map is an IFαg^{**} -closed map but converse may not be true.
Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy closed mapping. Let A be an IFCS in X. Since f is an intuitionistic fuzzy closed mapping, $f(A)$ is an IFCS in Y. Since every IFCS is an IFαg^{**}CS, $f(A)$ is an IFα^{**}GCS in Y. Hence f is an IFαg^{**} -closed mapping.

6.4 Example Let X = {a, b}, Y = {x, y} and A= < x, (0.5, 0.6), (0.2, 0.1) >, B = < y, (0.5, 0.4), (0.2, 0.1) >. Then $\tau = \{0_\sim, A, 1_\sim\}$, $\sigma = \{0_\sim, B, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = x and f(b) = y. Then f is an IFαg^{**} -closed mapping but not intuitionistic fuzzy closed mapping .

6.5 Theorem Every intuitionistic fuzzy α-closed map is an IFαg^{**} -closed map but converse may not be true.
Proof: Let $f: (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy α-closed mapping. Let A be an IFCS in X. Since f is an intuitionistic fuzzy α-closed mapping, $f(A)$ is an IFαCS in Y. Since every IFαCS is an IFαg^{**}CS, $f(A)$ is an IFα^{**}GCS in Y. Hence f is an IFαg^{**} -closed mapping.

6.6 Example Let X = {a, b}, Y = {x, y} and A= < a, (0.5, 0.2), (0.5, 0.8) >, B = < x, (0.2, 0.4), (0.3, 0.6) >. Then $\tau = \{0_\sim, A, 1_\sim\}$, $\sigma = \{0_\sim, B, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = x and f(b) = y. Then f is an IFαg^{**}-closed mapping but not intuitionistic fuzzy α-closed mapping .

6.7 Theorem A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IFαg^{**} -closed mapping if and only if the image of each IFOS in X is an IFαg^{**}OS in Y.
Proof: Let A be an IFOS in X. This implies A^c is IFCS in X. Since f is an IFαg^{**} -closed mapping, $f(A^c)$ is an IFαg^{**}CS in Y. Since $f(A^c) = (f(A))^c$, $f(A)$ is an IFαg^{**}OS in Y.

The relation between various types of intuitionistic fuzzy closed mappings are given in the following diagram.

intuitionistic fuzzy closed mapping → intuitionistic fuzzy αg^{**}-closed mapping

intuitionistic fuzzy closed mapping → intuitionistic fuzzy α-closed mapping

intuitionistic fuzzy α-closed mapping → intuitionistic fuzzy αg^{**}-closed mapping

6.8 Theorem If $f:(X,\tau) \rightarrow (Y,\sigma)$ is an intuitionistic fuzzy closed map and $g:(Y,\sigma) \rightarrow (Z,\mu)$ is an IFαg^{**} -closed map, then $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is IFαg^{**} -closed mapping.
Proof: Let H be an IFCS of an IFTS (X,τ). Then $f(H)$ is IFCS of (Y,σ), because f is intuitionistic fuzzy closed map. Now $g \circ f(H) = g(f(H))$ is an IF$\alpha g^{**}CS$ in (Z,μ) because g is IFαg^{**} -closed map. Thus $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is IFαg^{**} -closed mapping.

6.9 Theorem Let $f:(X,\tau) \rightarrow (Y,\sigma)$ and $g:(Y,\sigma) \rightarrow (Z,\mu)$ are two intuitionistic fuzzy mappings such that their composition $g \circ f:(X,\tau) \rightarrow (Z,\mu)$ is IF -closed mapping. If f is intuitionistic fuzzy continuous and surjective, then g is IFαg^{**} -closed.
Proof: Let A be an IFCS of Y Since f is intuitionistic fuzzy continuous f^{-1}(A) is IFCS in X. Since $g \circ f$ is IFαg^{**} -closed, $g \circ f(f^{-1}$(A)) is intuitionistic fuzzy αg^{**}-closed in Z. That is $g(A)$ is IFαg^{**} -closed in Y, because f is surjective. Therefore g is IFαg^{**} -closed.

7. Intuitionistic Fuzzy αg^{**}- Homeomorphisms

In this section we introduced the concept of intuitionistic fuzzy αg^{**}- homeomorphisms and studied some of its properties.

7.1 Definition A bisection mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ is called an intuitionistic fuzzy αg^{**} -homeomorphism (IFαg^{**} -homeomorphism in short) if f and f^{-1} are IFαg^{**}-continuous mappings.

7.2 Example Let X = {a, b}, Y = {x, y} and A= < x, (0.2, 0.6), (0.6, 0.2) >, B = < y, (0.3, 0.7), (0.7, 0.3) >. Then $\tau = \{0_\sim, A, 1_\sim\}$, $\sigma = \{0_\sim, B, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by f(a) = x and f(b) = y. Then f is an intuitionistic fuzzy αg^{**}-homeomorphism.

Theorem 7.3: Every IF homeomorphism is an IFαg^{**} -homeomorphism but converse may not be true.
Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an IF homeomorphism. Then f and f^{-1} are IF continuous mappings. This implies f and f^{-1} are IFα^{**}G-continuous mappings. Hence f is IFαg^{**}-homeomorphism.

7.4 Example Let X = {a, b}, Y = {x, y} and A= < x, (0.5, 0.6), (0.3, 0.2) >, B = < y, (0.5, 0.6), (0.2, 0.2) >. Then $\tau = \{0_\sim, A, 1_\sim\}$, $\sigma = \{0_\sim, B, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by f(a) = x and f(b) = y.Then f is intuitionistic fuzzy αg^{**}-homeomorphism but not an IF homeomorphism.

7.5 Theorem Every IFα homeomorphism is an IFαg^{**} -homeomorphism but converse may not be true.
Proof: Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an IFα-homeomorphism. Then f and f^{-1} are IFα-continuous mappings. This implies f and f^{-1} are IFαg^{**} -continuous mappings. Hence f is IFαg^{**} -homeomorphism.

7.6 Example Let X = {a, b}, Y = {x, y} and A= < x, (0.2, 0.1), (0.2, 0.4) >, B = < y, (0.6, 0.8), (0.2, 0.2) >. Then $\tau = \{0_{\sim}, A, 1_{\sim}\}$, $\sigma = \{0_{\sim}, B, 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $\mathrm{f(a)} = \mathrm{x}$ and $\mathrm{f(b)} = \mathrm{y}$.Then f is an intuitionistic fuzzy αg^{**}-homeomorphism but not an IFα-homeomorphism.

7.7 Theorem Let $f: (X, \tau) \to (Y, \sigma)$ be a bijective mapping. If f is an IFαg^{**} -continuous mapping, then the following are equivalent.

(i) f is an IFαg^{**} - closed mapping
(ii) f is an IFαg^{**} - open mapping
(iii) f is an IFαg^{**} - homeomorphism.

Proof: (i)→(ii): Let $f: (X, \tau) \to (Y, \sigma)$ be a bijective mapping and let f be an IFαg^{**} - closed mapping. This implies $f^{-1}: (Y, \sigma) \to (X, \tau)$ is IFαg^{**} - continuous mapping. That is every IFOS in X is an IF$\alpha g^{**}OS$ in Y. Hence f is an IFαg^{**} - open mapping.
(ii)→(iii): Let $f: (X, \tau) \to (Y, \sigma)$ be a bijective mapping and let f be an IFαg^{**} - open mapping. This implies $f^{-1}: (Y, \sigma) \to (X, \tau)$ is IFαg^{**} - continuous mapping. But f is an IFαg^{**} - continuous mapping by hypothesis. Hence f and $f^{-1}$are IFαg^{**} -continuous mappings. Thus, f is IFαg^{**} -homeomorphism.
(iii)→(i): Let f be an IFαg^{**} -homeomorphism.That is f and $f^{-1}$are IFαg^{**} -continuous mappings. Sine every IFCS in X is an IF$\alpha g^{**}CS$ in Y, f is an IFαg^{**} - closed mapping.

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