On Decomposition of Intuitionistic Fuzzy Continuity

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Abstract:
There are various types of generalizations of intuitionistic fuzzy continuous functions in the development of intuitionistic fuzzy topology. In this paper, we obtain a decomposition of intuitionistic fuzzy continuity in intuitionistic fuzzy topological spaces by using intuitionistic fuzzy w-continuity and intuitionistic fuzzy rw-continuity.

Keywords: *Continuous function, Intuitionistic fuzzy w-closed set, Intuitionistic fuzzy rw-closed set, Intuitionistic fuzzy slc*-set, Intuitionistic fuzzy rslc*-set, Intuitionistic fuzzy slc*-intuitionistic fuzzy rslc**

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1. Introduction

In 1965, Zadeh [9] introduced fuzzy sets and in 1968, Chang [2] introduced fuzzy topology. After the introduction of fuzzy sets and fuzzy topology, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. The decomposition of intuitionistic fuzzy continuity is one of the many problems in intuitionistic fuzzy topology. V. Thiripurasundari and S. Murugesan [8] extend the notion of fuzzy p-set and fuzzy q-set to intuitionistic fuzzy p-set and intuitionistic fuzzy q-set and characterized intuitionistic fuzzy clopen sets and established decomposition of intuitionistic fuzzy totally continuous functions. In this direction we obtain a decomposition of intuitionistic fuzzy continuity in intuitionistic fuzzy topological space by using intuitionistic fuzzy w-continuity and intuitionistic fuzzy rw-continuity.

2. Preliminaries

Throughout this paper, (X, τ) or X denotes the intuitionistic fuzzy topological spaces (briefly IFTS). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequel.

2.1 Definition: [1]

Let X be a non-empty fixed set. An intuitionistic fuzzy set (IFS for short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ where the functions $\mu_A : X \rightarrow [0,1]$ and ν_A :

$X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

2.2 Definition: [1]

Let A and B be IFSs of the form $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$ and $B = \{<x, \mu_B(x), \nu_B(x)>/ x \in X\}$. Then

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ <x, \nu_A(x), \mu_A(x)>/ x \in X \}$,
(iv) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X \}$,
(v) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X \}$.

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x)\rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\rangle$. The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1\rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0\rangle / x \in X \}$ are respectively the empty set and the whole set of X.

2.3 Definition: [3]

An intuitionistic fuzzy topology (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X. The complement Ac of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

2.4 Definition: [3]

Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then

(i) $int(A) = \cup\{ G / G$ is an IFOS in X and $G \subseteq A \}$,
(ii) $cl(A) = \cap\{ K / K$ is an IFCS in X and $A \subseteq K \}$,
(iii) $cl(A^c) = (int(A))^c$,
(iv) $int(A^c) = (cl(A))^c$.

2.5 Definition: [4]

An IFS A of an IFTS (X,τ) is called an

(i) intuitionistic fuzzy semi closed set (IFSCS for short) if $int(cl(A)) \subseteq A$,
(ii) intuitionistic fuzzy semi open set (IFSOS for short) if $A \subseteq cl(int(A))$,

2.6 Definition: [7]

An IFS A of an IFTS (X, τ) is called an intuitionistic fuzzy regular semi open (IFRSOS for short) if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

2.7 Definition: [5]

An IFS A of an IFTS (X, τ) is called:

(i) Intuitionistic fuzzy w-closed (IFWCS for short) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy semi open.

(ii) Intuitionistic fuzzy w-open (IFWOS for short) if its complement A^c is intuitionistic fuzzy w-closed.

2.8 Remark: [5]
Every IFCS is an IFWCS but the converses may not be true in general.

2.9 Definition: [6]
An intuitionistic fuzzy set A of an IFTS (X, τ) is called an intuitionistic fuzzy rw-closed (IFRWCS for short) if $cl(A) \subseteq O$ whenever $A \subseteq O$ and O is intuitionistic fuzzy regular semi open in X.

2.10 Remark: [6]
Every IFCS is an IFRWCS but the converses may not be true in general.

2.11 Definition: [6]
An intuitionistic fuzzy set A of an IFTS (X, τ) is called intuitionistic fuzzy rw-open (IFRWOS for short) if and only if its complement A^c is intuitionistic fuzzy rw-closed.

2.12 Definition: [4]
Let (X, τ) and (Y, σ) be two IFTS's and let $f : X \to Y$ be a function. Then f is said to be an intuitionistic fuzzy continuous (IF continuous mapping for short) if the pre image of each intuitionistic fuzzy closed set in Y is an intuitionistic fuzzy closed set in X.

2.13 Definition: [5]
Let (X, τ) and (Y, σ) be two IFTS's and let $f : X \to Y$ be a function. Then f is said to be an intutionistic fuzzy w-continuous (IFW continuous mapping for short) if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy w-closed in X.

2.14 Remark: [5]
Every IF continuous mapping is an IFW continuous mapping but the converses may not be true in general.

2.15 Definition: [6]
Let (X, τ) and (Y, σ) be two IFTS's and let $f : X \to Y$ be a function. Then f is said to be an intutionistic fuzzy rw-continuous (IFRW continuous mapping for short) if the pre image of every intuitionistic fuzzy closed set in Y is intuitionistic fuzzy rw-closed in X.

2.16 Remark: [6]
Every IF continuous mapping is an IFRW continuous mapping but the converses may not be true in general.

3. Decomposition of Intuitionistic Fuzzy Continuity
In this section, we obtain a decomposition of intuitionistic fuzzy continuity in intuitionistic fuzzy topological spaces by using intuitionistic fuzzy w-continuity and intuitionistic fuzzy rw-continuity.

To obtain a decomposition of intuitionistic fuzzy continuity by using intuitionistic fuzzy w-continuity, we have introduce the notions of intuitionistic fuzzy slc^* -sets and intuitionistic fuzzy slc^* -continuous functions in intuitionistic fuzzy topological spaces and prove that a function is an intuitionistic fuzzy continuous if and only if it is both intuitionistic fuzzy w-continuous and intuitionistic fuzzy slc* -continuous.

3.1 Definition

A subset λ in a IFTS (X, τ) is called an intuitionistic fuzzy slc* -set (IFslc*-set for short) if $\lambda = \alpha \wedge \beta$, where α is an intuitionistic fuzzy semi open set and β is an intuitionistic fuzzy closed set in (X, τ).

3.2 Proposition

Every IFCS is an IFslc*-set but not conversely.

3.3 Remark

The concepts of IFWCS and IFslc*-sets are independent of each other.

3.4 Example

Let X= {a, b, c} and G_1= ⟨x, (0.3, 0.4, 0.4), (0.7, 0.6, 0.6)⟩ and G_2= ⟨x, (0.4, 0.5, 0.5), (0.6, 0.5, 0.5)⟩**.** Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on X and the IFS A= ⟨x, (0.7, 0.5, 0.5), (0.3, 0.5, 0.5)⟩ is an IFWCS but it is not an IFslc*-set in (X, τ).

3.5 Example

Let X= {a, b, c} and G_1= ⟨x, (0.4, 0.5, 0.6), (0.6, 0.5, 0.4)⟩ and G_2= ⟨x, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2)⟩**.** Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on X and the IFS A= ⟨x, (0.5, 0.6, 0.7), (0.5, 0.4, 0.3)⟩ is an IFslc*-set but it is neither IFCS nor IFWCS in (X, τ).

3.6 Proposition

Let (X, τ) be an IFTS. Then a subset λ of (X, τ) is an IFCS if and only if it is both IFWCS and IFslc*-set.

3.6.1 Proof

Necessity is trivial. To prove the sufficiency, assume that λ is both IFWCS and IFslc*-set. Then $\lambda = \alpha \wedge \beta$, where α is an IFSOS and β is IFCS in (X, τ). Therefore, $\lambda \leq \alpha$ and $\lambda \leq \beta$ and so by hypothesis, $cl(\lambda) \leq \alpha$ and $cl(\lambda) \leq \beta$. Thus $cl(\lambda) = \alpha \wedge \beta = \lambda$ and hence $cl(\lambda) = \lambda$ i.e., λ is IFCS in (X, τ).

3.7 Definition

A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy slc*-continuous (IFslc*- continuous for short) if for each IFCS μ of (Y, σ), $f^{-1}(\mu)$ is an IFslc*-set in (X, τ).

3.8 Example

Let X = {a, b, c}, Y = {u, v, w} and G_1= ⟨x, (0.4, 0.5, 0.6), (0.6, 0.5, 0.4)⟩, G_2= ⟨x, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2)⟩ and let G_3 = ⟨y, (0.5, 0.4, 0.3), (0.5, 0.6, 0.7)⟩. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFslc*-continuous mapping.

3.9 Remark

Every IF continuous mapping is an IFslc*-continuous but not conversely.

3.10 Remark

The concepts of an IFW continuity and IFslc*-continuity are independent of each other.

3.11 Example

Let X = {a, b, c}, Y = {u, v, w} and G_1=⟨x, (0.3, 0.4, 0.4), (0.7, 0.6, 0.6)⟩, G_2= ⟨x, (0.4, 0.5, 0.5), (0.6, 0.5, 0.5)⟩ and let G_3 = ⟨y, (0.3, 0.5, 0.5), (0.7, 0.5, 0.5)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFW continuous mapping but not an IFslc*-continuous mapping in X.

3.12 Example

Let X = {a, b, c}, Y = {u, v, w} and G_1= ⟨x, (0.4, 0.5, 0.6), (0.6, 0.5, 0.4)⟩, G_2= ⟨x, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2)⟩ and let G_3 = ⟨y, (0.5, 0.4, 0.3), (0.5, 0.6, 0.7)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFslc*-continuous mapping but not an IFW continuous mapping in X.

We have the following decomposition for intuitionistic fuzzy continuity.

3.13 Theorem

A function f: (X, τ) → (Y, σ) is IF continuous if and only if it is both IFW continuous and IFslc*-continuous.

3.13.1 Proof

Assume that f is an IF continuous mapping. Then by Remark 2.14 and Remark 3.9, f is both IFW continuous and IFslc*-continuous. Conversely, assume that f is both IFW continuous and IFslc*-continuous. Let μ be an IFS of (Y, σ). Then $f^{-1}(\mu)$ is both IFWCS and IFslc*-set. By Proposition 3.6, $f^{-1}(\mu)$ is an IFCS in (X, τ) and so f is an IF continuous.

Now to obtain a decomposition of intuitionistic fuzzy continuity by using IFRW continuity, we have introduce the notions of intuitionistic fuzzy rslc* -sets and intuitionistic fuzzy rslc* -continuous functions in intuitionistic fuzzy topological spaces and prove that a function is an intuitionistic fuzzy continuous if and only if it is both intuitionistic fuzzy rw-continuous and intuitionistic fuzzy rslc* -continuous.

3.14 Definition

A subset γ in a IFTS (X, τ) is called an intuitionistic fuzzy rslc* -set (IFrslc*-set for short) if γ = α ∧ β, where α is an intuitionistic fuzzy regular semi open set and β is an intuitionistic fuzzy closed set in (X, τ).

3.15 Proposition

Every IFCS is an IFrslc*-set but not conversely.

3.16 Remark

The concepts of IFRWCS and IFrslc*-sets are independent of each other.

3.17 Example

Let X= {a, b, c} and G_1= ⟨x, (0.3, 0.4, 0.4), (0.7, 0.6, 0.6)⟩ and G_2= ⟨x, (0.4, 0.5, 0.5), (0.6, 0.5, 0.5)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} is an IFT on X and the IFS A= ⟨x, (0.7, 0.5, 0.5), (0.3, 0.5, 0.5)⟩ is an IFRWCS but it is not an IFrslc*-set in (X, τ).

3.18 Example

Let X= {a, b, c} and G_1= ⟨x, (0.4, 0.5, 0.2), (0.6, 0.5, 0.8)⟩ and G_2= ⟨x, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} is an IFT on X and the IFS A= ⟨x, (0.5, 0.5, 0.7), (0.5, 0.5, 0.3)⟩ is an IFrslc*-set but it is neither IFCS nor IFRWCS in (X, τ).

3.19 Proposition

Let (X, τ) be an IFTS. Then a subset γ of (X, τ) is an IFCS if and only if it is both IFRWCS and IFrslc*-set.

3.19.1 Proof

Necessity is trivial. To prove the sufficiency, assume that γ is both IFRWCS and IFrslc*-set. Then $\gamma = \alpha \wedge \beta$, where α is an IFRSOS and β is IFCS in (X, τ). Therefore, $\gamma \leq \alpha$ and $\gamma \leq \beta$ and so by hypothesis, $cl(\gamma) \leq \alpha$ and $cl(\gamma) \leq \beta$. Thus $cl(\gamma) = \alpha \wedge \beta = \gamma$ and hence $cl(\gamma) = \gamma$ i.e., λ is IFCS in (X, τ).

3.20 Definition

A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be intuitionistic fuzzy rslc*-continuous (IFrslc*-continuous for short) if for each IFCS μ of (Y, σ), $f^{-1}(\mu)$ is an IFrslc*-set in (X, τ).

3.21 Example

Let X = {a, b, c}, Y = {u, v, w} and $G_1 = \langle x, (0.4, 0.5, 0.2), (0.6, 0.5, 0.8)\rangle$, $G_2 = \langle x, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2)\rangle$ and let $G_3 = \langle y, (0.5, 0.5, 0.3), (0.5, 0.5, 0.7)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF rslc*-continuous mapping.

3.22 Remark

Every IF continuous mapping is an IFrslc*-continuous but not conversely.

3.23 Remark

The concepts of an IFRW continuity and IFrslc*-continuity are independent of each other.

3.24 Example

Let X = {a, b, c}, Y = {u, v, w} and $G_1 = \langle x, (0.3, 0.4, 0.4), (0.7, 0.6, 0.6)\rangle$, $G_2 = \langle x, (0.4, 0.5, 0.5), (0.6, 0.5, 0.5)\rangle$ and let $G_3 = \langle y, (0.3, 0.5, 0.5), (0.7, 0.5, 0.5)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFRW continuous mapping but not an IFrslc*-continuous in X.

3.25 Example

Let X = {a, b, c}, Y = {u, v, w} and $G_1 = \langle x, (0.4, 0.5, 0.2), (0.6, 0.5, 0.8)\rangle$, $G_2 = \langle x, (0.6, 0.7, 0.8), (0.4, 0.3, 0.2)\rangle$ and let $G_3 = \langle y, (0.5, 0.5, 0.3), (0.5, 0.5, 0.7)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFrslc*-continuous mapping but not an IFRW continuous in X.

We have the following decomposition for intuitionistic fuzzy continuity.

3.26 Theorem

A function f : $(X, \tau) \rightarrow (Y, \sigma)$ is IF continuous if and only if it is both IFRW continuous and IFrslc*-continuous.

3.27 Proof

Assume that f is an IF continuous mapping. Then by Remark 2.16 and Remark 3.22, f is both IFRW continuous and IFrslc*-continuous. Conversely, assume that f is both IFRW continuous and IFrslc*-continuous. Let μ be an IFS of (Y, σ). Then $f^{-1}(\mu)$ is both IFRWCS and fuzzy IFrslc*-set. By Proposition 3.19, $f^{-1}(\mu)$ is an IFCS in (X, τ) and so f is an IF continuous.

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